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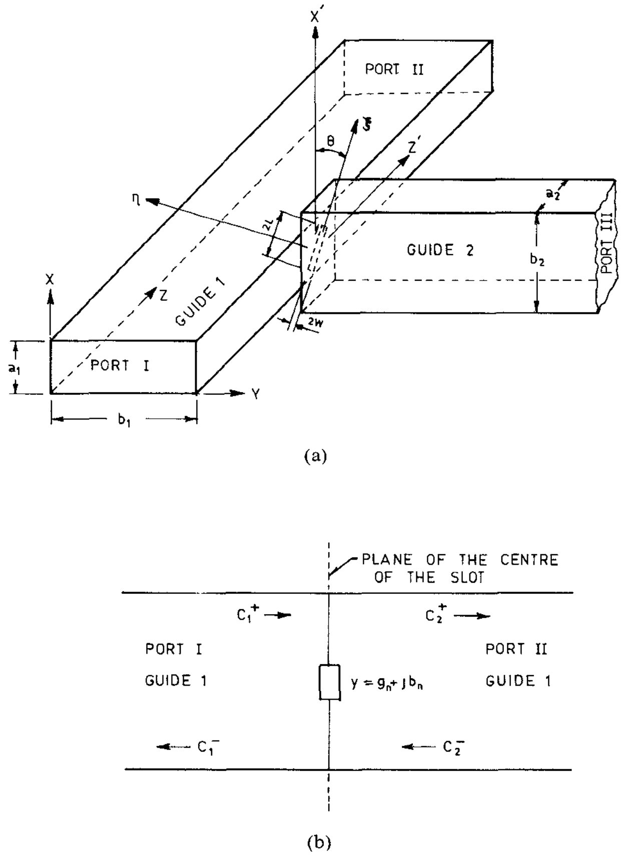

Fig. 1 (a) Coplanar $E-H$ plane T-junction. (b) Equivalent network representing impedance loading in guide 1.

Analysis of Coplanar $E-H$ Plane T-Junction Using Dissimilar Rectangular Waveguides

B. N. DAS, G S. N. RAJU, AND AJOY CHAKRABORTY

***Abstract*—An analysis of a T-junction which differs from conventional H-plane T-junctions in that the T arm is rotated by 90° and coupling takes place through an inclined slot is presented. Since use of standard X-band waveguides result in such a T-junction operating above 11.7 GHz, nonstandard waveguide dimensions have been considered to bring down the operating frequency to 9.375 GHz. The effect of a change of the broad dimension of the primary feed waveguide on the resonant conductance is evaluated. The variations of resonant length with the angle of inclination of the slot, and coupling with frequency, are presented.**

I. INTRODUCTION

Investigations of H-plane T-junctions have already been reported [1]. In the present paper, thorough analytical investigations have been carried out on a T-junction (Fig. 1(a)) in which the coupling slot is in the narrow wall of the primary guide and the narrow dimension of the coupled guide is oriented along the axis of the primary guide. As a result, the E field of the coupled guide and the H field of the primary guide are coplanar and hence this type of T-junction is designated as a coplanar $E-H$ plane T-junction.[1] The power is coupled to the T-arm through an inclined slot in the narrow wall of the primary guide. It can be noted that in this type of T-junction no power can be coupled using either a longitudinal or a vertical slot. When such a coupler is made using standard X-band waveguides the maximum slot length which can be obtained is 14 mm with an inclination of 45° and the resonant frequency is around 12 GHz.

In the present work, investigations have been carried out to find the waveguide dimensions necessary for slots which resonate around 9.4 GHz. Further investigations have also been carried out to keep the normalized slot conductance loading on the primary guide as low as 0.01. This impedance is expressed in terms of self-reaction and discontinuity in modal current. Evaluation of self-reaction in the coupled guide employs TE and TM mode fields instead of the hybrid mode field used earlier [1]. To obtain the self-reaction in the primary guide, the magnetic current in the inclined slot is resolved into transverse and longitudinal components.

Computations have been carried out to obtain the waveguide dimensions for which a low value of normalized slot conductance is obtained at a resonant frequency around 9.375 GHz. Computed results of the various parameters of engineering importance, e.g., resonant slot length, slot conductance, and coupling, are presented.

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[1]The Editor has kindly brought to the notice of the authors that this type of structure was reported by W. H. Watson in Fig. 34 of his paper "Resonant slots," *JIEE* (London), vol. 93, pt 3A, pp 747–777, 1946

II. General Analysis

Fig. 1(a) represents a T-junction in which the two rectangular waveguides forming the junction have different broad and narrow wall dimensions. Coupling takes place through an inclined slot in the narrow wall of guide 1. The relation between the variables along the coordinate axes shown in Fig. 1(a) is of the form

$$Z = Z' + a_2/2 = \xi \sin\theta - \eta \cos\theta + a_2/2$$
$$X = X' + b_2/2 = \xi \cos\theta + \eta \sin\theta + b_2/2. \tag{1}$$

The unit vectors through the coordinate axes are related by the following expressions:

$$\bar{u}_z = \bar{u}_\xi \sin\theta - \bar{u}_\eta \cos\theta$$
$$\bar{u}_x = \bar{u}_\xi \cos\theta + \bar{u}_\eta \sin\theta. \tag{2}$$

Following the formulation suggested in the literature [3] and using relations (1) and (2), the expressions for the normalized modal vector functions for electric ($\bar{e}^e_{mn}$) and magnetic ($\bar{e}^m_{mn}$) fields in guide 2 are found. The modal voltages V^e_{mn} and V^m_{mn} for the electric and magnetic fields, respectively, are given by [3]

$$V^e_{mn} = \int_{-W}^{W}\int_{-L}^{L} \bar{E}_t \cdot \bar{e}^e_{mn}\, d\xi\, d\eta \tag{3}$$

$$V^m_{mn} = \int_{-W}^{W}\int_{-L}^{L} \bar{E}_t \cdot \bar{e}^m_{mn}\, d\xi\, d\eta \tag{4}$$

where $\bar{E}_t$ is the electric field in the aperture plane of the slot and can be assumed to be of the form

$$\bar{E}_t = \bar{u}_\eta E_0 \sin K(L - |\xi|) \tag{5}$$

where $K = 2\pi/\lambda$, λ being the wavelength, E_0 is the maximum electric field in the slot and $2L$ is the slot length. Using (1)–(5) and the expressions for $\bar{e}^e_{mn}$ and $\bar{e}^m_{mn}$, the expressions for the modal voltages are obtained.

The transverse component of the magnetic field in the $y = 0$ plane of guide 2 is of the form

$$\bar{H}_t = \sum\sum \left[(Y_0)^e_{mn} V^e_{mn} \bar{h}^e_{mn} + (Y_0)^m_{mn} V^m_{mn} \bar{h}^m_{mn} \right] \tag{6}$$

where $(Y_0)^e_{mn}$ and $(Y_0)^m_{mn}$ are the characteristic admittances of the TE and TM modes, respectively, and $\bar{h}^e_{mn}$ and $\bar{h}^m_{mn}$ are the modal vector functions of the transverse component of magnetic field.

The total magnetic current in the aperture plane of the slot considering its image in the ground plane is given by

$$\bar{M} = 2\bar{E}_t \times \bar{u}_y. \tag{7}$$

The self-reaction of this magnetic current $\bar{M}$ in guide 2 is of the form

$$\langle a, a\rangle_2 = -\iint \bar{H}_t \cdot \bar{M}\, d\xi\, d\eta. \tag{8}$$

The impedance loading in guide 1 due to the power flow and energy storage in guide 2 is given by

$$Z_2 = -\langle a, a\rangle_2 / II \tag{9}$$

where I is the discontinuity in modal current in guide 1.

Using (5)–(9) and the expression for I suggested in the literature [1], [4], the expression for Z_2 is found to be of the form

where

$$(Y_0)^e_{mn} = \frac{(\gamma_2)_{mn}}{j\mu\omega} \qquad (Y_0)^m_{mn} = \frac{j\omega\epsilon}{(\gamma_2)_{mn}}$$

$$(\gamma_2)_{mn} = \sqrt{\left(\frac{m\pi}{b_2}\right)^2 + \left(\frac{n\pi}{a_2}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2}.$$

Y_{01} and β_{01} are, respectively, the characteristic admittance and propagation constants.

In the above expression, the term for $m = 0$, $n = 1$ contributes to the real part of the impedance and the other terms contribute to he imaginary part of the impedance.

In order to reduce the complexity of the derivation of the expression for the energy storage in the primary guide, the magnetic current in the aperture plane of the slot is resolved into transverse (length $= 2L\cos\theta$, width $= 2W\cos\theta$) and longitudinal (length $= 2L\sin\theta$, width $= 2W\sin\theta$) components.

The impedance loading on the primary guide due to self-reaction of the transverse component is of the form

$$Z_3 = -\langle a, a\rangle_3 / II \tag{11}$$

and that due to longitudinal component is of the form

$$Z_1 = -\langle a, a\rangle_1 / II. \tag{12}$$

Using the method suggested in the literature [2], [3], the expression for self-reaction due to the transverse component is obtained as

$$\langle a, a\rangle_3 = \frac{2jK^2V_0^2\cos^2\theta}{\omega\mu a_1 b_1 W} \sum_m^\infty \sum_n^\infty \frac{\epsilon_m}{(\gamma_1)^2_{mn}} \frac{1}{K^2 - (n\pi/a_1)^2} \cdot \cos^2 m\pi \sin^2\frac{n\pi}{2}\left[\cos(n\pi L\cos\theta/a_1) - \cos(KL\cos\theta)\right]^2 \cdot \left[2\cos\theta + \left(e^{-2(\gamma_1)_{mn}W\cos\theta}\right)\Big/(\gamma_1)_{mn}W - \frac{1}{(\gamma_1)_{mn}W}\right] \tag{13}$$

where

$$(\gamma_1)_{mn} = \sqrt{\left(\frac{m\pi}{b_1}\right)^2 + \left(\frac{n\pi}{a_1}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2} \qquad V_0 = 2WE_0.$$

In the above summation, the terms for $m = 0$, $n = 0$ and $m = 1$, $n = 0$ are excluded.

Self-reaction of the longitudinal component is of the same form as [1, eq. (7)] with the exception that a is replaced by b_1 and b is replaced by a_1, d is replaced by $2W\sin\theta$ and L is replaced by $L\sin\theta$, and V_m is replaced by $V_0\sin\theta$.

An expression for the total normalized impedance seen by the primary guide is

$$Z/Z_0 = (Z_1 + Z_2 + Z_3)/Z_0 \tag{14}$$

where Z_0 ($= \mu\omega/\beta_{01}$) is the impedance of the dominant mode in

$$Z_2 = \frac{a_1 b_1^3 \beta_{01}^2 \left\{(\beta_{01}\sin\theta)^2 - K^2\right\}^2 \cdot \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left[(Y_0)^e_{mn}(V^e_{mn})^2 + (Y_0)^m_{mn}(V^m_{mn})^2\right]}{4K^2 Y_{01}^2 V_0^2 \sin^2\theta\, \pi^2 \left\{\dfrac{\sin(W\beta_{01}\cos\theta)}{W\beta_{01}\cos\theta}\right\}^2 \cdot \left\{\cos(\beta_{01}L\sin\theta) - \cos KL\right\}^2} \tag{10}$$

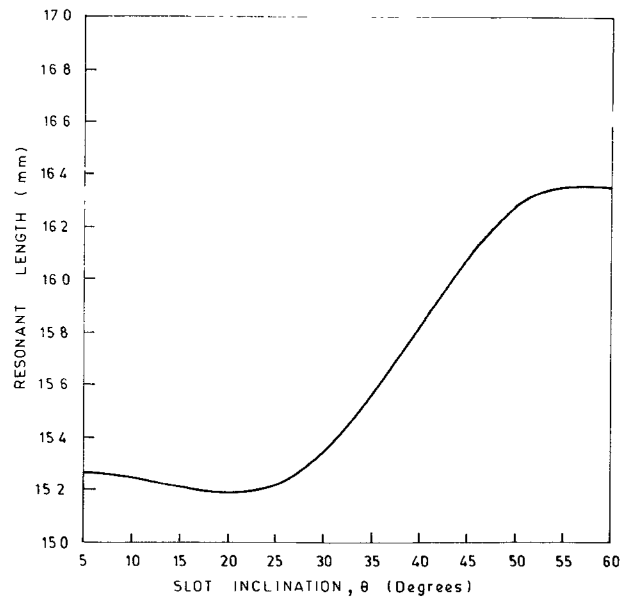

Fig 2. Variation of resonant length with the inclination of slot (θ) for $2W = 0.2$ mm, $f = 9.375$ GHz, $a_1 = a_2 = 1.52$ cm, and $b_1 = b_2 = 2.286$ cm.

guide 1. The corresponding normalized admittance is given by

$$Y = g_n + jb_n = 1/z. \tag{15}$$

The equivalent circuit with the incident and reflected wave amplitudes at the input and output is shown in Fig. 1(b). For a matched guide $c_2^- = 0$. Using the expression for the transmission matrix parameters in accordance with the formulation suggested by Collin [5], it can be shown that the expression for the coupling is of the form

$$C_{\text{dB}} = 10\log\left[4g_n/\left\{(2+g_n)^2 + b_n^2\right\}\right]. \tag{16}$$

Levy [6] has given complete information for wall thickness correction applicable to circular apertures and rectangular slots. for long slots, however, the first-order correction term [1], which can be applied without appreciable deterioration in accuracy, is $-8.686\alpha t$, where $\alpha = \sqrt{(\pi/2L)^2 - K^2}$ and t is the wall thickness of the waveguide.

III. Numerical Results

It has been found that the loading on the primary guide can be reduced if θ is reduced. The reduction in θ to low values is permitted if the narrow dimension of both the waveguides is increased. The value of θ can be reduced to 5° if $a_1 = a_2 = 1.52$ cm and $b_1 = b_2 = 2.286$ cm. The variation of corresponding resonant length with θ for a frequency of 9.375 GHz is presented in Fig. 2. Using (10)–(15), the variations of g_n and b_n are evaluated as a function of frequency. The results are substituted to compute the variation of coupling with frequency for θ ranging from 5° to 30°. The data on coupling, estimated after taking the correction due to wall thickness into account, are presented in Fig. 3.

Increase of b_1 does not disturb the constraints dictated by the design considerations for the array and permits the reduction of g_r (resonant conductance) below 0.01. It is, however, not advisable to increase the narrow wall dimension beyond 1.50 cm. For $\theta = 10°$, $2L = 1.516$ cm, $\lambda = 3.2$ cm, $a_1 = a_2 = 1.50$ cm, $b_2 = 2.286$ cm, g_r is evaluated for b_1 ranging from 2.286 cm to 3.0 cm and the results are presented in Fig. 4.

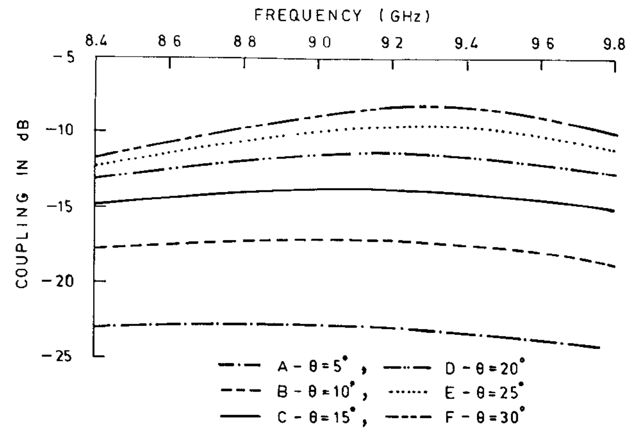

Fig 3. Variation of coupling with frequency for $2W = 0.2$ mm, $a_1 = a_2 = 1.52$ cm, and $b_1 = b_2 = 2.286$ cm (curve A for $\theta = 5°$, B for $\theta = 10°$, C for $\theta = 15°$, D for $\theta = 20°$, E for $\theta = 25°$, F for $\theta = 30°$).

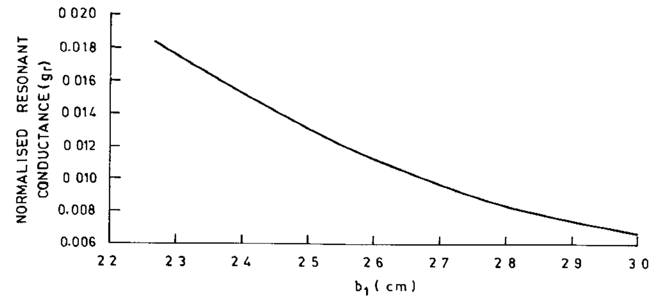

Fig. 4. Variation of resonant conductance (g_r) with broad dimension of the guide 1 (b_1) for $2W = 0.2$ mm, $2L \approx 1.516$ cm (resonant length), $f = 9.375$ GHz, $a_1 = a_2 = 1.50$ cm, and $b_2 = 2.286$ cm.

IV. Discussion

Results of the analysis reveal that it is possible to obtain resonance at $\lambda = 3.2$ cm in the junction having a waveguide with internal dimensions $a = 1.52$ cm, $b = 2.286$ cm. Resonant conductance has been brought down to the order of 0.005 by suitable reduction of slot inclination. Because of the increase in narrow wall dimension, the frequency range of operation is restricted to 8.4–9.8 GHz. In this frequency range, the conductance is quite high compared to susceptance, and this variation with frequency remains more or less stationary up to $\theta = 20°$. Further reduction in g_r has been achieved by increasing the broad dimension of the feed waveguide.

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